OCR
Oxford Cambridge and RSA

Wednesday 18 May 2016 – Morning

A2 GCE MATHEMATICS

4729/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4729/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 A car of mass 1400 kg is travelling on a straight horizontal road against a constant resistance to motion of 600 N. At a certain instant the car is accelerating at $0.3\,\mathrm{m\,s^{-2}}$ and the engine of the car is working at a rate of 23 kW.

(i) Find the speed of the car at this instant. **[3]**

Subsequently the car moves up a hill inclined at 10° to the horizontal at a steady speed of $12\,\mathrm{m\,s^{-1}}$. The resistance to motion is still a constant 600 N.

(ii) Calculate the power of the car's engine as it moves up the hill. **[3]**

2 *A* and *B* are two points on a line of greatest slope of a plane inclined at 55° to the horizontal. *A* is below the level of *B* and *AB* = 4 m. A particle *P* of mass 2.5 kg is projected up the plane from *A* towards *B* and the speed of *P* at *B* is $6.7\,\mathrm{m\,s^{-1}}$. The coefficient of friction between the plane and *P* is 0.15. Find

(i) the work done against the frictional force as *P* moves from *A* to *B*, **[3]**

(ii) the initial speed of *P* at *A*. **[4]**

3

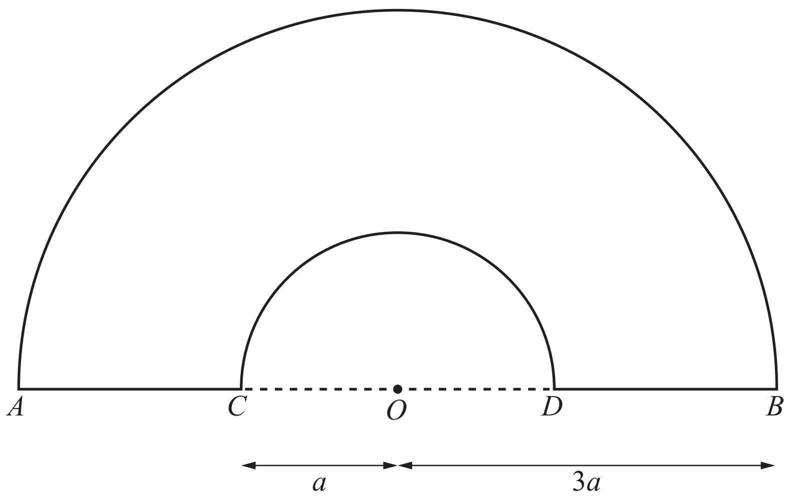


Fig. 1

A uniform lamina *ABDC* is bounded by two semicircular arcs *AB* and *CD*, each with centre *O* and of radii 3*a* and *a* respectively, and two straight edges, *AC* and *DB*, which lie on the line *AOB* (see Fig. 1).

(i) Show that the distance of the centre of mass of the lamina from *O* is $\frac{13a}{3\pi}$. **[5]**

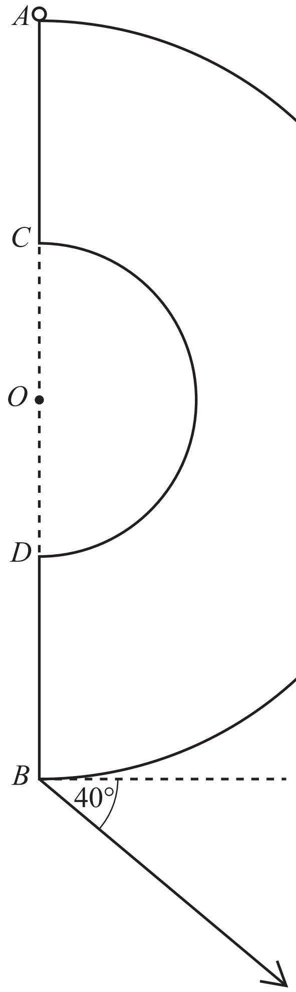


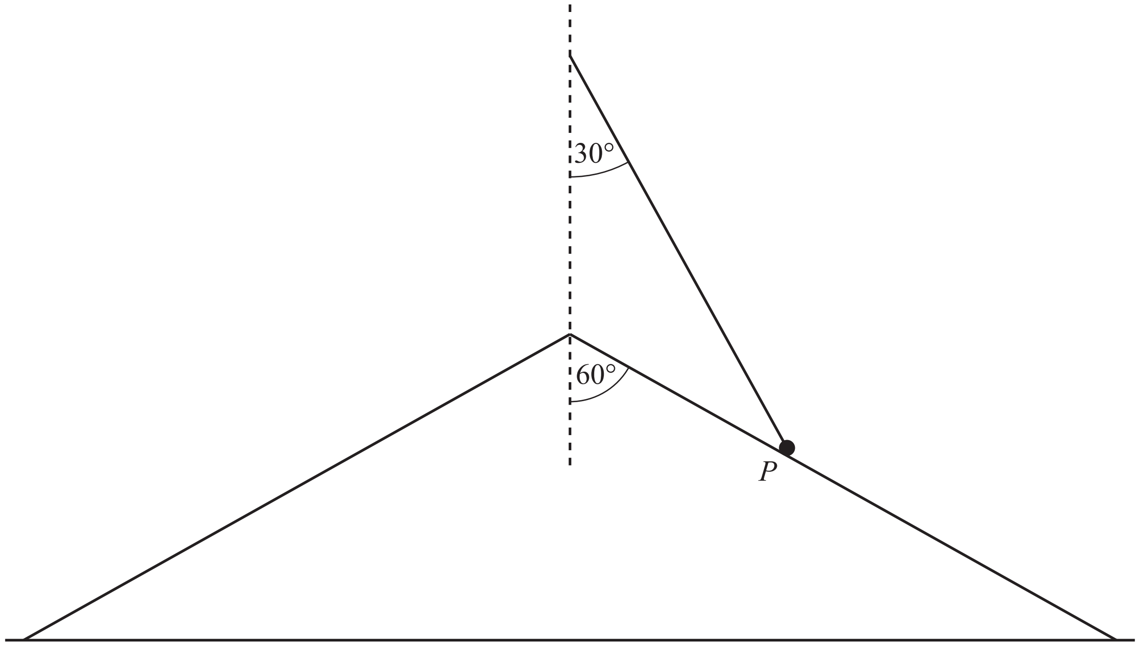
Fig. 2

The lamina has mass 3 kg and is freely pivoted to a fixed point at A. The lamina is held in equilibrium with AB vertical by means of a light string attached to B. The string lies in the same plane as the lamina and is at an angle of 40° below the horizontal (see Fig. 2).

(ii) Calculate the tension in the string. **[3]**

(iii) Find the direction of the force acting on the lamina at A. **[4]**

4



A smooth solid cone of semi-vertical angle 60° is fixed to the ground with its axis vertical. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point vertically above the vertex of the cone. P rotates in a horizontal circle on the surface of the cone with constant angular velocity ω. The string is inclined to the downward vertical at an angle of 30° (see diagram).

(i) Show that the magnitude of the contact force between the cone and the particle is $\frac{1}{6}m(2\sqrt{3}g - 3a\omega^2)$. **[6]**

(ii) Given that $a = 0.5$ m and $m = 3.5$ kg, find, in either order, the greatest speed for which the particle remains in contact with the cone and the corresponding tension in the string. **[3]**

5 A uniform ladder AB, of weight W and length $2a$, rests with the end A in contact with rough horizontal ground and the end B resting against a smooth vertical wall. The ladder is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{12}{13}$. A man of weight $6W$ is standing on the ladder at a distance x from A and the system is in equilibrium.

(i) Show that the magnitude of the frictional force exerted by the ground on the ladder is $\frac{5W}{24}\left(1 + \frac{6x}{a}\right)$. **[5]**

The coefficient of friction between the ladder and the ground is $\frac{1}{3}$.

(ii) Find, in terms of a, the greatest value of x for which the system is in equilibrium. **[3]**

The bottom of the ladder A is moved closer to the wall so that the ladder is now inclined at an angle α to the horizontal. The man of weight $6W$ can now stand at the top of the ladder B without the ladder slipping.

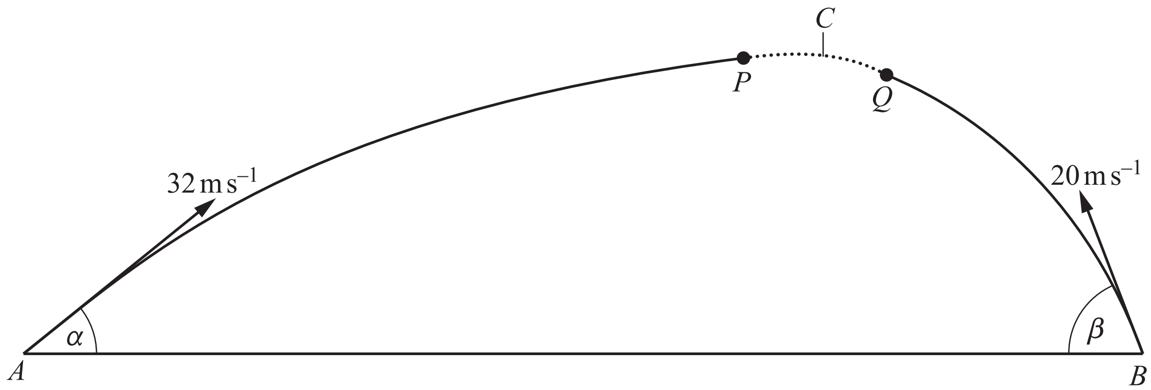
(iii) Find the least possible value of $\tan\alpha$. **[3]**

6 The masses of two particles A and B are 4 kg and 3 kg respectively. The particles are moving towards each other along a straight line on a smooth horizontal surface. A has speed $8\,\text{m s}^{-1}$ and B has speed $10\,\text{m s}^{-1}$ before they collide. The kinetic energy lost due to the collision is 121.5 J.

(i) Find the speed and direction of motion of each particle after the collision. **[8]**

(ii) Find the coefficient of restitution between A and B. **[2]**

7



A particle P is projected with speed $32\,\text{m s}^{-1}$ at an angle of elevation α, where $\sin\alpha = \frac{3}{5}$, from a point A on horizontal ground. At the same instant a particle Q is projected with speed $20\,\text{m s}^{-1}$ at an angle of elevation β, where $\sin\beta = \frac{24}{25}$, from a point B on the same horizontal ground. The particles move freely under gravity in the same vertical plane and collide with each other at the point C at the instant when they are travelling horizontally (see diagram).

(i) Calculate the height of C above the ground and the distance AB. **[4]**

Immediately after the collision P falls vertically. P hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height 5 m above the ground.

(ii) Given that the mass of P is 3 kg, find the magnitude and direction of the impulse exerted on P by the ground. **[4]**

The coefficient of restitution between the two particles is $\frac{1}{2}$.

(iii) Find the distance of Q from C at the instant when Q is travelling in a direction of 25° below the horizontal. **[9]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance
1	(i)		Driving force = $\frac{23000}{v}$	B1	
			$\frac{23000}{v} - 600 = 1400(0.3)$	M1	Attempt at N2L with 3 terms; allow D – 600 = 1400(0.3)
			$v = 22.5\text{ m s}^{-1}$	A1 [3]	$v = 22.54901...$; allow $^{1150}/_{51}$
	(ii)		$D - 600 - mg\sin 10 = 0$	M1	Attempt at N2L with three terms $(D = 2982.452998)$; *g* needed
			$P = (\text{cv}(D))(12)$	M1	Use of $P = Dv$
			$P = 35.8\text{ kW}$ or 35800 W	A1 [3]	$P = 35789.43597$
2	(i)		$Fr = 0.15(2.5g\cos 55)$	M1	Resolving perpendicular and use of $Fr = \mu R$
				M1	Use of work done = friction $\times$ distance *AB*, using their friction; if combined with work done by other forces M0
			Work done $= 8.43\text{ N m}$	A1 [3]	Work done = 8.4315736…
	(ii)		$2.5g(4\sin 55)$	B1	PE term; 80.27690034
			$\pm\left(\frac{1}{2}(2.5)u^2 - \frac{1}{2}(2.5)(6.7)^2\right)$	B1	Change in KE
			$\frac{1}{2}(2.5)u^2 - 2.5g(4\sin 55) - \frac{1}{2}(2.5)(6.7)^2 = \text{cv}(8.43)$	M1	Use of work – energy principle (4 terms); dimensionally correct
			$u = 10.8\text{ m s}^{-1}$	A1 [4]	$u = 10.763678...$
	OR		+/-2.5*a* = -0.15(2.5*g*cos55) – 2.5*g*sin55	M1* A1	Use of N2L with 3 terms (a = -8.87074739…. : allow *a* positive)
			$6.7^2 = u^2 + 2 \times 4 \times a$ $u = 10.8\text{ m s}^{-1}$	M1 dep* A1	Method to find *u* using their deceleration, leading to u > 6.7 *u* = 10.763678….

Question			Answer	Marks	Guidance
3	(i)		CoM of one semicircular lamina is $\frac{4a}{\pi}$, $\frac{4a}{3\pi}$	B1	oe, may be unsimplified, allow $^{4r}/_{3\pi}$
				M1	Table of values idea
			$-\left(\frac{1}{2}\pi a^2\right)\left(\frac{4a}{3\pi}\right)+\left(\frac{1}{2}\pi(3a)^2\right)\left(\frac{12a}{3\pi}\right)$	A1	
			$=\left(\frac{1}{2}\pi(3a)^2-\frac{1}{2}\pi a^2\right)x_G$	A1	
			$x_G=\frac{13a}{3\pi}$	A1	AG Correctly shown
				[5]	
	(ii)			M1	Moments equation in terms of *a* and *T*, with correct number of terms, dimensionally correct, resolving on *T* side of equation; if conflicting evidence about point taking moments mark to benefit of candidate.
			$3g\left(\frac{13a}{3\pi}\right)=(T\cos 40)(6a)$	A1	
			$T=8.82\text{ N}$	A1	$T=8.822960...$
				[3]	
	(iii)		$X=T\cos 40$	B1	ft candidates value of T if substituted $(X=6.758779...)$
			$Y=3g+T\sin 40$	B1	ft candidates value of T if substituted $(Y=35.071289...)$
			$\tan\theta=\frac{35.0712...}{6.7587...}$	M1	Any relevant angle
			$\theta=79.1$ <u>above</u> horizontal (to the left)	A1	$\theta=79.0919...$ (10.9 to the <u>upward</u> vertical); above or upwards may be implied by a correct diagram.
				[4]	
	OR		$x^2=(3g)^2+T^2-2\times 3g\times T\times\cos 130$	M1A1ft	
			Use of sine rule to find either of the missing angles	M1	
			$\theta=79.1$ <u>above</u> horizontal (to the left)	A1	$\theta=79.0919...$ (10.9 to the <u>upward</u> vertical); above or upwards may be implied by a correct diagram.

Question			Answer	Marks	Guidance
4	(i)			M1*	Resolving vertically (3 terms)
			$T\cos 30 + R\sin 60 = mg$	A1	
				M1*	Resolving horizontally (3 terms); an r used where r is not just a
			$T\sin 30 - R\cos 60 = m(a\sin 30)\omega^2$	A1	
				M1 dep*	Eliminating T and solve for R in terms of m, g, a and ω
			$R = \frac{1}{6}m\left(2\sqrt{3}g - 3a\omega^2\right)$	A1	**AG** Correctly shown
				[6]	
	(ii)		For using $R = 0$ to attempt to find either v or T	M1	Or attempt to find ω
			$T = \frac{mg}{\cos 30} = 39.6$	A1	39.606228…
			$\omega^2 = \frac{T}{ma}$, $v = 1.19\ \text{m s}^{-1}$	A1	1.1893309…
				[3]	
5	(i)		$Fr = R_w$	B1	Resolving horizontally
				M1	Moments about A all forces accounted for
			$Wa\cos\theta + 6Wx\cos\theta = 2aR_w\sin\theta$	A1	A1 for at least two terms correct, may involve θ
				A1	Fully correct equation without θ
			$Fr = \frac{5W}{24}\left(1 + \frac{6x}{a}\right)$	A1	**AG** Correctly shown
				[5]	
	OR		$R_g = 7W$	B1	Resolving vertically
				M1	Moments about B all forces accounted for
			$Wa\cos\theta + 6W(2a - x)\cos\theta + 2aFr\sin\theta = 2aR_g\cos\theta$	A1A1	A1 for two terms correct
			$Fr = \frac{5W}{24}\left(1 + \frac{6x}{a}\right)$	A1	**AG** Correctly shown

Question			Answer	Marks	Guidance
	(ii)		$R_g = 7W$ $\frac{5W}{24}\left(1+\frac{6x}{a}\right)=\frac{1}{3}(7W)$ $x = 1.7a$	B1 M1 A1 **[3]**	Resolving vertically (maybe seen in (i) and used in (ii)) Use of $Fr = \mu R$ with candidates R_g ; allow ≤, <, >, ≥ $17a/10$, $51a/30$; must be = or ≤
	(iii)		$\frac{Wa\cos\alpha + 6W(2a)\cos\alpha}{2a\sin\alpha} = \frac{1}{3}(7W)$ (oe for moments about B) $\tan\alpha = \frac{39}{14}$	M1 A1 A1 **[3]**	Setting $x = 2a$ in their dimensionally correct moments equation **and** use of $Fr = \mu R$ allow ≤, <, >, ≥ $\tan\alpha = 2.7857142...$; must be = or ≥
6	(i)		$4(8)+3(-10) = 4v_A + 3v_B$ $\frac{1}{2}(4)(8)^2 + \frac{1}{2}(3)(10)^2 - \frac{1}{2}(4)v_A{}^2 - \frac{1}{2}(3)v_B{}^2 = 121.5$ $v_A = -5.5$ $(v_A = 6.0714...)$ so speed of A is 5.5 (m s^{-1}) $v_B = 8$ $(v_B = -7.428...)$ so speed of B is 8 (m s^{-1}) Both particles are moving in the reverse direction to their original motion	M1* A1 M1* A1 M1 dep* A1 A1 A1 **[8]**	Attempt at use of conservation of momentum Attempt at use of KE(before) – KE (after) = 121.5 Obtaining quadratic equation in either v_A or v_B $\left(7v_B{}^2 - 4v_B - 416 = 0,\ \ 28v_A{}^2 - 16v_A - 935 = 0\right)$ and attempt to solve quadratic for either v_A or v_B cao; must be positive cao; must be positive Or an equivalent statement consistent with their v_A and v_B; left and right not sufficient without a diagram; moving away from each other needs a diagram also
	(ii)		$v_A - v_B = -e(8-(-10))$ $e = 0.75$	M1 A1 **[2]**	Attempt at use of coefficient of restitution, right way round, v_A and v_B substituted

Question			Answer	Marks	Guidance
7	(i)		$\left(32\left(\frac{3}{5}\right)\right)^2 - 2(9.8)h = 0$	M1	Use of $v^2 = u^2 + 2as$ with $v = 0$ $\left(\text{or}\left(20(24/25)\right)^2 - 2(9.8)h = 0\right)$
			$h = 18.8\text{m}$	A1	$h = 18.808163...$ or $\frac{4608}{245}$
			$t = \frac{32}{9.8}\left(\frac{3}{5}\right)(=1.96)$	B1	1.9591836…, $\frac{96}{49}$
			$AB = \left(32\left(\frac{4}{5}\right) + 20\left(\frac{7}{25}\right)\right)t = 61.1\text{m}$	B1	61.126530… or $\frac{14976}{245}$
				[4]	
	OR		For last two marks may use range formula	B1	For correct expression for finding half the range for one particle (50.155.. or 10.971…)
			61.1m	B1	61.126530… or $\frac{14976}{245}$
	(ii)		Speed of P at impact $\sqrt{2(9.8)\left(\frac{4608}{245}\right)}$	B1ft	cv(h) from (i); (19.2)
			Speed of P after impact $\sqrt{2(9.8)(5)}$	B1	$7\sqrt{2} = 9.899494937$
			$I = 3\left(7\sqrt{2} - \left(-\frac{96}{5}\right)\right)$	M1	Attempt at Impulse = change in momentum
			$I = 87.3$ N s, upwards	A1	Must be positive; 87.298484… ;must have direction stated.
				[4]	

Question		Answer	Marks	Guidance
	(iii)		M1*	Attempt at use of coefficient of restitution, component of 32 and 20 needed;
		$0-v_B=-\frac{1}{2}\left(32\left(\frac{4}{5}\right)-\left(-20\left(\frac{7}{25}\right)\right)\right)$		
		$v_B=15.6$	A1	$\frac{78}{5}$
		Vertical component of speed $v_B \tan 25$	B1*	7.274399467 (soi);
		$\left(v_B \tan 25\right)^2=2(9.8)y$	M1*	Use of $v^2=u^2+2as$ vertically with $u=0$ to find vertical displacement with their vertical speed component
		$y=2.70$	A1	2.6998412…
		$t=\frac{v_B \tan 25}{9.8}$	B1*	$t=0.74228565...$
		$x=v_B t$	B1*	$x=11.5796562...$
		$QC=\sqrt{x^2+y^2}$	M1 dep*	
		$QC=11.9\text{m}$	A1	11.890230…
			[9]	Allow wrong v_B for max M1A0B1M1A0B1B1M1A0 If v_B "found" not using restitution then M0A0B1M1A0B1B1M0A0 max